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Boson–fermion model beyond the mean-field approximation

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Abstract. A model of hybridized bosons and fermions is studied beyond the mean-field approximation. The divergent boson self-energy at zero temperature makes the Cooper pairing of fermions impossible. The frequency and momentum dependence of the self-energy and the condensation temperature T_c of initially localized bosons are calculated analytically. The value of the boson condensation temperature T_c is below 1 K and the long-wave spectrum is strongly damped which rules out the boson–fermion model with the initially localized bosons as a phenomenological explanation of high-temperature superconductivity. The intra-cell density–density fermion–boson interaction dominates in the fermion self-energy. The model represents a dirty normal metal, in which localized bosons play the role of impurities.

1. Introduction

Many superconducting and normal state properties of perovskites favour a charged $2e$ Bose liquid of small bipolarons as a plausible microscopic model of their ground state [1]. In particular, Bose-liquid features are clearly verified by the λ -like specific heat near the transition [2], the characteristic shape of the upper critical field [3], by the ‘boomerang’ behaviour of T_c and the London penetration depth with doping [4], explained recently [5].

In a multi-band system a mixture of bipolarons and electrons is feasible, with bipolarons formed in a narrow band (the bandwidth $W \ll E_F$) and almost free fermions, with a large Fermi energy E_F . If they interact with each other exclusively via the density–density interaction the effect of fermions is that of screening the Coulomb boson–boson interaction. Hence, an acoustic gapless plasmon mode is expected in the entire temperature regime including the superfluid state while the fermionic component remains normal [6]. The Bose–Einstein condensation temperature is expected to be about that of an ideal Bose gas.

On the other hand Friedberg and Lee [7], Ranninger and collaborators [8] and several other authors [9] studied bosons *hybridized* with fermions, a so-called *boson–fermion model* (BFM). The BFM has been motivated by the difficulty of accommodating a stable mobile bosonic field because of the allegedly strong Coulomb repulsion. Then the underlying mechanism for superconductivity has been assumed to be through the reaction $e + e \rightarrow \phi \rightarrow e + e$ involving a *virtual* $2e$ boson ϕ . Because of this transition, it has been claimed that ‘the zero momentum virtual bosons force the two e ’s to have equal and opposite momenta, forming a Cooper pair’ [7]. The studies carried out on BFM with initially localized bosons also showed a superconducting ground state, ‘controlled by the condensation of the bosons and a concomitantly driven BSC-like state of the fermionic subsystem’ with the BCS-like gap in the electron spectrum [8]. It has been claimed [7–9] that BFM gives the possibility of achieving large values of critical temperature.

In this paper we study the boson–fermion model beyond the mean-field approach, applied in [7–9], by taking into account the boson self-energy and the intra-cell density–density repulsive interaction. We provide a rigorous proof that the Cooper pairing of fermions is impossible at any value of the repulsion. The ground state of BFM is essentially the same as that of the boson–fermion mixture with the normal fermionic component discussed earlier by us [6]. The boson energy spectrum, damping and the density of states as well as the condensation temperature T_c are calculated. The intra-cell correlations dominate in the fermion self-energy. The role of the hybridization interaction is shown to be negligible both for T_c and for the fermion damping.

2. No Cooper pairing

BFM is defined by the following Hamiltonian [7, 8]:

$$H = \sum_{k,s=\uparrow,\downarrow} \xi_k c_{k,s}^\dagger c_{k,s} + \sum_q \omega_0(\mathbf{q}) b_q^\dagger b_q + \frac{v}{\sqrt{N}} \sum_{q,k} (b_q^\dagger c_{k+q,\uparrow} c_{-k,\downarrow} + \text{HC}) \quad (1)$$

where ξ_k is the fermionic energy with respect to the chemical potential E_F , $\omega_0(\mathbf{q}) = E_i - 2E_F + q^2/2M$ is the bare boson energy with E_i the energy level of the doubly occupied $2e$ sites. The bare boson mass M can be infinite for initially localized bosons [8]. The boson–fermion hybridization interaction $v \simeq \Gamma^2/|U|$ is of second order with respect to the single-electron interband hybridization Γ . BFM is applied if the attractive on-site interaction $U < 0$, responsible for the boson formation and the Fermi energy E_F are large compared with Γ , so $v \lll |U|, E_F$. N is the number of sites (cells) in the normalized volume and $\hbar = c = 1$. Here we do not discuss the attraction mechanism that would account for the formation of bosons. Some discussion of the mechanisms can be found in [1, 8, 9].

In all real-life systems the Coulomb repulsion exists, $V_c > 0$, which is fairly represented by the Hamiltonian

$$H_c = \frac{V_c}{N} \sum_{k,k',q,s,s'} [c_{k+q,s}^\dagger c_{k'-q,s'}^\dagger c_{k',s'} c_{k,s} + 2c_{k+q,s}^\dagger c_{k,s} b_{k'-q}^\dagger b_{k'}] \quad (2)$$

which describes the intra-cell correlations. If we want to keep within the limits of this particular BFM we shall take $V_c = 0$. However, the strong inequality $V_c \gg v$ is normally satisfied.

The criterion for Cooper-pair formation in a Fermi liquid lies in the existence of a nontrivial solution to the linearized BCS equation [10]

$$\Delta(p) = - \int dp' V(p, p') G(p') G(-p') \Delta(p'). \quad (3)$$

Under this condition the two-particle vertex part, figure 1(a), has a pole in the Cooper channel. One can identify Δ with the superconducting order parameter and the temperature T_x at which the nontrivial solution to (3) appears as the superconducting transition temperature for fermions. $V(p, p')$ is defined in the sense that it cannot be divided into two parts by cutting two parallel fermion propagators $G(p)$; $p \equiv (\mathbf{k}, i\omega_n)$ is the momentum and the fermionic Matsubara frequency $\omega_n = \pi T(2n + 1)$, $n = 0, \pm 1, \pm 2, \dots$, so that $\int dp' \equiv T \sum_{k,n}$. In the leading order in v the irreducible interaction $V(p, p')$ is given by figure 1(b),

$$V(p, p') = \frac{v^2}{N} D_0(0, 0) + \frac{V_c}{N} \quad (4)$$

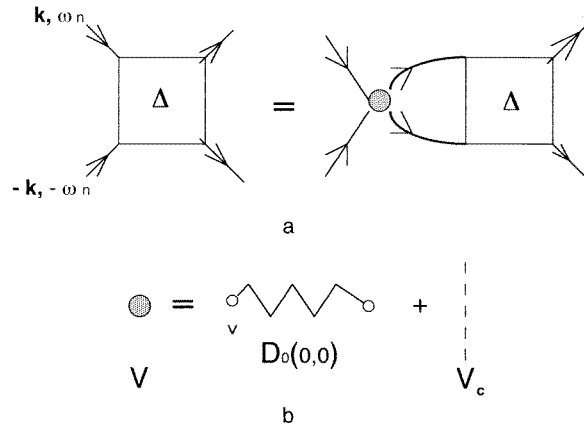


Figure 1. Two-particle vertex part for (a) the Cooper channel and (b) irreducible fermion–fermion interaction.

where

$$D_0(\mathbf{q}, \Omega_n) = \frac{1}{i\Omega_n - \omega_0(\mathbf{q})} \quad (5)$$

is the *bare* boson temperature Green’s function with $\Omega_n = 2\pi Tn$. The physical (i.e. renormalized) fermion $G(\mathbf{k}, \omega_n)$ and boson $D(\mathbf{q}, \Omega_n)$ Green’s functions satisfy the sum rule

$$\frac{2T}{N} \sum_{\mathbf{q}, n} e^{i\Omega_n \tau} D(\mathbf{q}, \Omega_n) = \frac{T}{N} \sum_{\mathbf{k}, n} e^{i\omega_n \tau} G(\mathbf{k}, \omega_n) - n_e \quad (6)$$

which determines the chemical potential E_F of the system. Here n_e is the carrier density per cell and $\tau = +0$.

By the use of (4) and (5) the BCS equation is reduced to a simple form

$$1 = \left(\frac{v^2}{N\omega_0(0)} - \frac{V_c}{N} \right) \int dp' G(p')G(-p') \quad (7)$$

which can be readily solved by replacing the exact fermion propagator for the bare one, $G(p) \simeq (i\omega_n - \xi_k)^{-1}$. Then the critical temperature is given by (for $V_c = 0$)

$$T_x \simeq 1.14E_F \exp\left(-\frac{\omega_0(0)}{v^2N(0)}\right) \quad (8)$$

where $N(0) \simeq 1/E_F$ is the density of fermionic states. This is a mean-field result, which led several authors [7–9] to the conclusion that BFM represents a high-temperature superconductor if $\omega_0(0)$ is low enough. However, one has to recognize that the bare boson energy $\omega_0(0)$ has no physical meaning and hence the expression (8) is meaningless. On the other hand the ‘physical’ (i.e. renormalized) zero-momentum boson energy $\omega(0)$ is well defined. It should be positive or zero according to the sum rule, equation (6) in the renormalized model:

$$\omega(0) = \omega_0(0) + \Sigma_b(0,0) \geq 0. \quad (9)$$

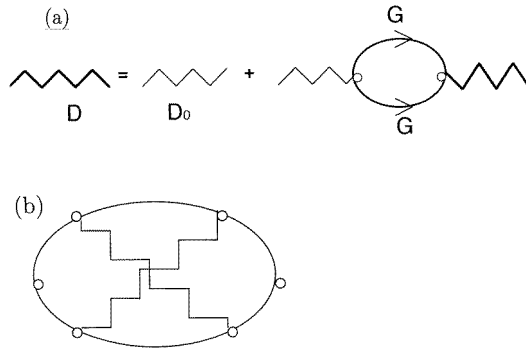


Figure 2. (a) Boson Green's function and (b) the lowest order 'crossing' diagram.

Here $\Sigma_b(\mathbf{q}, \Omega_n)$ is the boson self-energy given by figure 2 [11] as

$$\Sigma_b(\mathbf{q}, \Omega_n) = -\frac{v^2}{N} T \sum_{\mathbf{k}', n'} G(\mathbf{k}' + \mathbf{q}, \omega_{n'} + \Omega_n) G(-\mathbf{k}', -\omega_{n'}). \quad (10)$$

This equation is almost exact. The sixth and higher orders in v 'crossing diagrams', figure 2(b) are as negligible as $v/E_F \ll 1$. It allows us to express the nonphysical bare energy $\omega_0(0)$ via the 'physical' $\omega(0)$ as

$$\omega_0(0) = \omega(0) + \frac{v^2}{N} \int dp' G(p') G(-p'). \quad (11)$$

Then the BCS equation, equation (3) takes the following form:

$$\frac{\omega(0)}{\omega(0) + (v^2/N) \int dp' G(p') G(-p')} = -\frac{V_c}{N} \int dp' G(p') G(-p'). \quad (12)$$

It has no solution because $\omega(0) \geq 0$ and $V_c > 0$. Therefore we conclude that there is no pairing of fermions ($\Delta = 0$) in the boson–fermion model at any temperature in sharp contrast to the mean-field result of [7–9]. This conclusion is exact because no assumptions have been made as far as the fermion Green's function $G(p)$ is concerned. In particular, taking into account the fermion self-energy due to the hybridization interaction v or due to the fermion–fermion and fermion–boson repulsion (see below) does not affect our conclusion.

One can erroneously believe that the renormalized boson propagator $D(0,0)$ rather than the bare one $D_0(0,0)$ should be applied in the expression for the irreducible vertex $V(p, p')$ (figure 1(b), equation (4)), in which case the physical zero-momentum energy $\omega(0)$ would appear in the expression for T_x , equation (8). This is incorrect because replacing D_0 for D in the Cooper channel, figure 1(a), leads to a *double counting* of the hybridization interaction. The same Cooperon diagram is responsible for the boson self-energy, figure 2(a). In other words the bare bosons only contribute to the Cooper channel. They are never condensed ($\omega_0(0) > 0$ for any value of v) and, therefore cannot induce the superconducting state of the fermionic subsystem. From a pedagogical point of view it is interesting to note that a similar *double-counting* problem appears in the calculation of the response function of condensed charged bosons. As has been discussed in [12] one should use the free-particle propagator rather than the renormalized one to derive the textbook expression [13] for the boson dielectric response function.

The authors of [7–9] applying the mean-field approach failed to recognize the divergence of the boson self-energy at zero temperature. It diverges logarithmically, so the bare

boson energy is infinite at zero temperature and the pairing interaction ($\sim v^2 N(0)/\omega_0(0)$) is zero. The divergent boson self-energy fully compensates the divergent Cooperon diagram. Friedberg and Lee [7] discussed the self-energy effect, missing, however, the Fermi-distribution function in their out-of-place expression for the boson self-energy, which does not respect the Pauli principle (equation (1.15) of their paper). As a result, they failed to notice the ‘infrared’ collapse of their theory.

3. Condensation of strongly damped bosons

While the fermionic subsystem remains normal at any temperature the bosons can be condensed at some finite temperature T_c . If their bare mass M is sufficiently low, T_c is given by the ideal Bose-gas formula [6],

$$T_{c0} \simeq 3.3 \frac{n_B^{2/3}}{Ma^2} \quad (13)$$

where $n_B = (n_e - n_F)/2$ is the boson density per cell, n_F is the fermion density and a is the lattice constant. With their computer calculations of the boson damping Ranninger *et al* [11] argue that the initially *localized* bosons with $M = \infty$ change over into free-particle-like propagating states as the temperature is lowered. However, since the mean-field arguments are incorrect and the numerical results are restricted by a finite q -space and temperature region the conclusion on the possibility of the superfluidity in BFM with $M = \infty$ is far from evident.

In this section we suggest the analytical calculation of the boson self-energy and show that the long-wave bosonic excitations are strongly damped. Their condensation is possible at very low temperature (< 1 K) in the localized rather than propagating state.

The condensation temperature T_c is given by the sum rule, equation (6) at $\omega(0) = 0$

$$-\frac{T}{N} \sum_{q,n} \frac{e^{i\Omega_n \tau}}{i\Omega_n + \Sigma_b(0,0) - \Sigma_b(\mathbf{q}, \Omega_n)} = n_B. \quad (14)$$

By the use of the analytical properties of the boson self-energy the sum on the left-hand side is replaced by an integral as

$$\int_0^\infty \frac{\rho(z)}{\exp(z/T_c) - 1} dz = n_B \quad (15)$$

where

$$\rho(z) = \frac{1}{\pi N} \sum_{\mathbf{q}} \frac{\gamma(\mathbf{q}, z)}{[z - \omega(\mathbf{q}, z)]^2 + \gamma^2(\mathbf{q}, z)} \quad (16)$$

is the boson density of states. In the leading order in v one can use the bare fermionic propagator in (10) to calculate $\Sigma_b(\mathbf{q}, \Omega_n)$ as

$$\Sigma_b(\mathbf{q}, \Omega_n) = -\frac{v^2}{N} \sum_{\mathbf{k}} \frac{\tanh(\xi_{\mathbf{k}}/2T) + \tanh(\xi_{\mathbf{k}+\mathbf{q}}/2T)}{\xi_{\mathbf{k}} + \xi_{\mathbf{k}+\mathbf{q}} - i\Omega_n}. \quad (17)$$

The analytical continuation to real frequencies is then

$$\omega(\mathbf{q}, z) \equiv \Re \Sigma_b(\mathbf{q}, z) - \Sigma_b(0,0) = \frac{z_c}{4} \int_{-\infty}^{\infty} dx \tanh\left(\frac{qv_F x}{4T_c}\right) \left[\ln \frac{|x-1-z/qv_F|}{|x+1-z/qv_F|} + \frac{2}{x} \right] \quad (18)$$

for the real part, and

$$\gamma(\mathbf{q}, z) \equiv \Im \Sigma_b(\mathbf{q}, z) = \pi z_c \frac{T_c}{q v_F} \ln \left(\frac{\cosh(z + q v_F / 4 T_c)}{\cosh(z - q v_F / 4 T_c)} \right) \quad (19)$$

for the damping. Here $z_c = v^2 N(0)$ and v_F is the Fermi velocity. By the use of these equations we obtain the following asymptotic behaviour of the boson energy and of the damping in the long-wave $q \ll q_c = 4 T_c / v_F$ and low energy $z \ll q v_F$ limit:

$$\omega(q) \simeq \frac{q^2}{2 M^*} \quad (20)$$

$$\gamma \simeq z \frac{\pi z_c}{8 T_c} \quad (21)$$

where the inverse bosonic 'mass' is determined by

$$\frac{1}{M^*} = \frac{z_c v_F^2}{6 \pi^2 T_c^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}. \quad (22)$$

Substitution of these expressions into (16) yields the square root asymptotic behaviour of the boson density of states at low energies $z \rightarrow 0$

$$\rho(z) \sim \sqrt{z} \quad (23)$$

which makes the integral in (15) convergent and the condensation possible. However, the damping in this long-wave region is large. On the mass surface $z = \omega(q)$ we find

$$\frac{\gamma}{\omega(q)} = \frac{\pi z_c}{8 T_c} \simeq 1 \quad (24)$$

because $T_c \leq z_c$ as we show below. Therefore the long-wave spectrum (equation (20)) as well as the bosonic mass M^* due to hybridization have no physical meaning, and a finite T_c does not signal the occurrence of superfluidity.

The strongly damped part of the spectrum has a negligible weight in the total number of bosonic states due to a very small value of q_c compared with the reciprocal lattice constant, $q_c \ll 1/a$. The inequality $q \gg q_c$ is fulfilled practically in the whole Brillouin zone. In this case by the use of (18) and (19) we find

$$\omega(q) \simeq z_c \ln \frac{q}{q_c} \quad (25)$$

and

$$\gamma = z \frac{\pi z_c}{2 q v_F}. \quad (26)$$

In this region the damping is small, $\gamma/\omega(q) \ll 1$, and the energy spectrum, equation (25) is well defined. It is practically dispersionless, so the boson density of states is well represented by the δ -function

$$\rho(z) \simeq \delta(z - z_c). \quad (27)$$

Now the critical temperature is readily obtained from (15) by the use of (27) as

$$T_c = \frac{v^2 N(0)}{\ln(1 + 1/n_B)}. \quad (28)$$

The condensation itself does not lead to superfluidity. Because of the damping, equation (21), the spectrum cannot be described with the momentum, and the bosons condense into a localized state rather than the extended one. We can compare

T_c , equation (28) with the Bose–Einstein condensation temperature T_{c0} , equation (13), determined with the finite bare boson mass $M \simeq |U|/W^2a^2$. Taking $n_B \simeq 1$ one obtains

$$\frac{T_c}{T_{c0}} \simeq \frac{\Gamma^4}{E_F|U|W^2} \ll 1 \quad (29)$$

because in any realistic case $\Gamma \leq W$. As an example, if one believes [8, 9] that localized bipolarons in YBCO are associated with the Cu–O chains and mobile single-particle states are associated with the Cu–O planes, the hybridization matrix element Γ is proportional to the chain–plane overlap integral and is clearly of the same order or even less than the intra-chain hopping integral ($\sim W$). As far as the polaronic reduction of the bandwidth is concerned, by the use of the displacement canonical transformation and the Holstein model [1] one can readily show that the reduction factor is precisely the same for both the bare bipolaronic band (the bandwidth $\sim 1/Ma^2$) and for the hybridized one (the bandwidth of the order of v^2/E_F). Moreover, for intersite bipolarons and dispersive phonons the polaron orthogonality blocking of the (bi)polaron tunnelling is less significant than in the Holstein model as discussed recently by us [5]. At the same time the hybridization matrix element Γ remains suppressed to the same extent as in the dispersionless Holstein model [14]. Therefore, in general the orthogonality blocking (phonon overlap) reduces the ratio T_c/T_{c0} even further. As a result, the relative value of the critical temperature of the (localized) boson–fermion model is small. The absolute value is very low as well. By taking $E_F \sim |U| \simeq 1$ eV and $\Gamma \leq 0.1$ eV one estimates $T_c \simeq \Gamma(\Gamma^3/E_F|U|^2) \leq 1$ K. This estimate and the strong damping of the long-wave part of the spectrum rule out BFM as an explanation of high-temperature superconductivity. The effect of hybridization on the Bose–Einstein condensation appears to be negligible. As we show below the effect of hybridization on the normal state fermion spectra is negligible as well.

4. Fermion self-energy

The effect of hybridization on the normal state fermion spectrum in the BFM with initially localized bosons was discussed by several authors (see, for example, [15]). It was shown that the hybridization leads to a one-particle self-energy equivalent of the one of the marginal Fermi liquid. If the temperature is above $T_c \sim 1$ K the fermion self-energy $\Sigma_f^h(\mathbf{k}, \omega_n)$ due to hybridization is described well by the following expression (see figure 3) [11]

$$\Sigma_f^h(\mathbf{k}, \omega_n) = -\frac{v^2}{N} T \sum_{\mathbf{k}', \omega_n'} \frac{1}{[i(\omega_{n'} + \omega_n) - \omega(0)][i\omega_{n'} - \xi_{\mathbf{k}'}]}. \quad (30)$$

In this temperature range the finite boson bandwidth and the damping can be neglected while the use of the renormalized zero momentum boson energy $\omega(0) > 0$ rather than $\omega_0(0)$ prohibits the violation of the sum rule, equation (6). The sum over frequencies is expressed as

$$\Sigma_f^h(\mathbf{k}, \omega_n) = \frac{v^2}{N} \sum_{\mathbf{k}'} \frac{\coth(\omega(0)/2T) - \tanh(\xi_{\mathbf{k}'}/2T)}{i\omega_n - \omega(0) + \xi_{\mathbf{k}'}}. \quad (31)$$

Continued to the real frequencies this expression yields the following result for the imaginary part:

$$\Im \Sigma_f^h(z) = \text{sign}(z) \pi z_c \left[\coth \frac{\omega(0)}{2T} - \tanh \frac{\omega(0) - z}{2T} \right]. \quad (32)$$

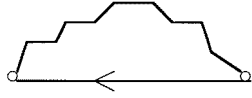


Figure 3. Hybridization contribution to the fermion self-energy.

Then the fermion lifetime due to hybridization is expressed using the sum rule as

$$\Im \Sigma_f^h(z) = 4\pi z_c \frac{n_B(1+n_B)}{1+2n_B - \tanh(z/2T)}. \quad (33)$$

If the temperature $T \ll \omega(0)$ and the fermion is far away from the Fermi surface ($z > \omega(0)$) the lifetime is

$$\Im \Sigma_f^h = 2\pi z_c \quad (34)$$

which is the Fermi golden rule for spontaneous transitions to the empty local pair states. At the Fermi surface ($z = 0$) the lifetime is proportional to the boson density

$$\Im \Sigma_f^h = 4\pi z_c \frac{n_B(1+n_B)}{1+2n_B} \quad (35)$$

because the fermion needs some energy to annihilate.

We compare the hybridization lifetime, equation (35) with the damping due to boson density fluctuations coupled with the fermion density, equation (2). As far as the direct repulsion between fermions is concerned (the first term in (2)), its contribution is negligible near the Fermi surface if $V_c \ll E_F$, which is assumed here. The corresponding leading contribution from H_c to the fermion self-energy is then presented in figure 4 and expressed as

$$\Sigma_f^c(\mathbf{k}, \omega_n) = \frac{4V_c^2}{N} T \sum_{q, n'} \frac{\Pi_b(\mathbf{q}, \Omega_{n'})}{i(\omega_n - \Omega_{n'}) - \xi_{\mathbf{k}-\mathbf{q}}} \quad (36)$$

where

$$\Pi_b(\mathbf{q}, \Omega_n) = \frac{1}{N} \sum_{q'} \frac{\coth(\omega(\mathbf{q}' + \mathbf{q})/2T) - \coth(\omega(\mathbf{q}')/2T)}{i\Omega_n - \omega(\mathbf{q} + \mathbf{q}') + \omega(\mathbf{q})} \quad (37)$$

is the boson response function. There is only static response in the limit $M \rightarrow \infty$ of initially localized bosons, so that

$$\Pi_b(\mathbf{q}, \Omega_n) = \frac{n_B(1+n_B)}{T} \delta_{\Omega_n, 0}. \quad (38)$$

Substituting this expression into (36) we finally obtain

$$\Sigma_f^c(z) = i \operatorname{sign}(z) 4\pi V_c^2 N(0) n_B(1+n_B). \quad (39)$$

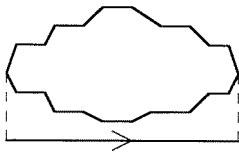


Figure 4. Fluctuation contribution to the fermion self-energy.

The ratio of two lifetimes at the Fermi surface is given by

$$\frac{\Im \Sigma_f^h}{\Im \Sigma_f^c} = \frac{v^2}{V_c^2(1 + 2n_B)} \quad (40)$$

which is about 10^{-4} for the appropriate values of v and V_c .

It appears that the boson density fluctuations lead to an attractive interaction between two fermions, figure 5, so that the total pairing potential is now

$$V(p, p') = -\frac{v^2}{N\omega_0(0)} + \frac{V_c}{N} - \frac{4V_c^2}{NT} n_B(1 + n_B) \delta_{\omega_n, \omega_{n'}}. \quad (41)$$

The BCS equation takes the following form:

$$\Delta(\omega_n) = (\lambda - \mu)\pi T_c \sum_{n'} \frac{\Delta(\omega_{n'})}{|\tilde{\omega}_{n'}|} + 4\pi V_c^2 N(0) n_B(1 + n_B) \frac{\Delta(\omega_n)}{|\tilde{\omega}_n|} \quad (42)$$

where $\tilde{\omega}_n = \omega_n + \text{sign}(\omega_n)4\pi V_c^2 N(0) n_B(1 + n_B)$ is the damped Matsubara frequency, $\lambda = v^2 N(0)/\omega_0(0)$ and $\mu = V_c N(0)$. Introducing a new order parameter $\tilde{\Delta}$ as

$$\tilde{\Delta} = \Delta(\omega_n) \left[1 - \frac{4\pi V_c^2 N(0) n_B(1 + n_B)}{|\tilde{\omega}_n|} \right] \quad (43)$$

we obtain the same BCS equation, equation (7) as in the absence of any density fluctuations

$$\tilde{\Delta} = (\lambda - \mu)\pi T_c \sum_{n'} \frac{\tilde{\Delta}}{|\omega_{n'}|}. \quad (44)$$

In this particular BFM it only has the trivial solution, $\tilde{\Delta} = 0$ because $\lambda \rightarrow 0$ when $T \rightarrow 0$, as explained above. As a result the density fluctuations of the bosonic field have no effect on T_c , which is a textbook result [10]. They play the same role in the BFM as the normal impurities in superconductors with no effect on the critical temperature in accordance with the Anderson theorem.

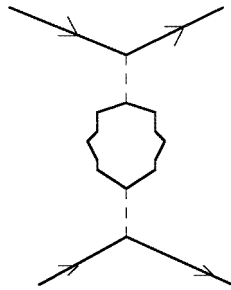


Figure 5. Effective attraction of fermions via boson density fluctuations.

5. Conclusion

Our study of the boson–fermion model beyond the mean-field approximation has shown that this approximation, which predicts a BCS-like fermionic superconductivity, is qualitatively wrong. The Cooper pairing of fermions due to their hybridization with the bosonic band is not possible. The long-wave bosons are strongly damped and their condensation temperature

is determined by the bare bosonic mass rather than by hybridization. The fermion self-energy due to the density–density coupling with bosons is larger by several orders of magnitude than that due to hybridization. BFM with initially localized bosons appears to be a normal dirty metal where bosons play the role of normal impurities.

Our results have a direct bearing upon the general problem of high-temperature superconductivity via the exchange interaction. In sharp contrast with the mean-field approach [7–9] the exact treatment of the boson–fermion model leads to the conclusion that this model cannot provide a high value of T_c . Boson–fermion hybridization plays no role either in the T_c value or in the fermion self-energy, which are determined by the bare effective mass of bosons and by the density–density fermion-boson coupling, respectively.

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References

- [1] Alexandrov A S and Mott N F 1994 *Rep. Prog. Phys.* **57** 1197; 1994 *High Temperature Superconductors and Other Superfluids* (London: Taylor and Francis)
- [2] Alexandrov A S and Ranninger J 1992 *Solid State Commun.* **81** 403
- [3] Alexandrov A S 1993 *Phys. Rev. B* **48** 10571
Alexandrov A S *et al* 1996 *Phys. Rev. Lett.* **76**
- [4] Uemura Y J 1995 *Polarons and Bipolarons in High- T_c Superconductors and Related Materials* ed E K H Salje, A S Alexandrov and W Y Liang (Cambridge: Cambridge University Press) pp 453–60
- [5] Alexandrov A S 1996 *Phys. Rev. B* **53** 1
- [6] Alexandrov A S and Khmelinin A B 1986 *Sov. Phys.–Solid State* **28** 1915
- [7] Friedberg R and Lee T D 1989 *Phys. Rev. B* **40** 6745
Friedberg R, Lee T D and Ren H C 1990 *Phys. Rev. B* **42** 4122
- [8] Ranninger J and Robin J M 1995 *Physica* **253C** 279
Micnas R, Ranninger J and Robaszkiewicz S 1990 *Rev. Mod. Phys.* **62** 113 and references therein
- [9] Bar-Yam Y 1991 *Phys. Rev. B* **43** 359, 1991 2601 and references therein
- [10] Abrikosov A A, Gor'kov L P and Dzyaloshinskii I E 1963 *Methods of Quantum Field Theory in Statistical Physics* (Englewood Cliffs, NJ: Prentice-Hall)
- [11] Note the opposite sign in our expression for the boson self-energy compared with that in J Ranninger *et al* 1995 *Phys. Rev. Lett.* **74** 4027. In their recent publication (1996 *Phys. Rev. B* **53** 11961) these authors corrected their error, but erroneously changed the sign in the electron self-energy.
- [12] Alexandrov A S and Beere W H 1995 *Phys. Rev. B* **51** 5887
- [13] Pines D and Nozières P 1966 *The Theory of Quantum Liquids* New York: Benjamin/Addison Wesley)
- [14] Ting C S, Talwar D N and Ngai K L 1980 *Phys. Rev. Lett.* **45** 1213
- [15] Alascio B R and Proetto C R 1990 *Solid State Commun.* **75** 217 and references therein